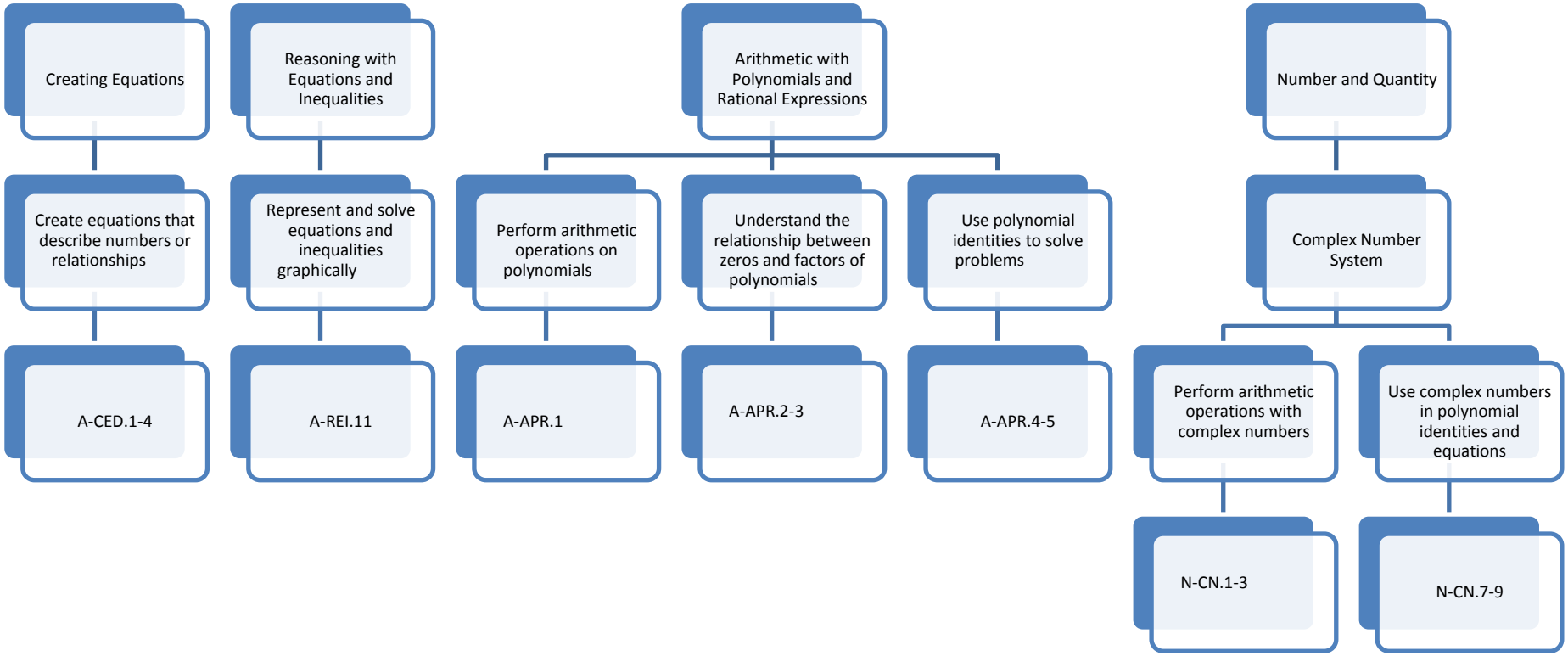
Honors Advanced Mathematics

Unit 1

Introduction and Preliminaries to Advanced Mathematics



Honors Advanced Math – UNIT 1
Introduction & Preliminaries to Advanced Mathematics

Critical Area:
Students use reasoning to analyze equations/ inequalities and develop strategies for solving them. Through reasoning students develop fluency writing, interpreting, analyzing and translating between various forms of linear equations and inequalities. By exploring a question about the world around them (mathematical modeling) and attempting to answer the question students expand the scope of algebraic operations to solve a wide variety of linear and quadratic real world problems. Students explain why the x-coordinates of the points where the graphs $y = f(x)$ and $y = g(x)$ intersects and explore cases involving polynomial, rational, absolute value, exponential, and logarithmic functions.

Students connect the polynomial operations with the background knowledge of the algorithms found in multi-digit integer operations. Students realize that the operations on rational expressions (the arithmetic of rational expressions) are governed by the same rules as the arithmetic of rational numbers. Students analyze the structure in expressions and write them in equivalent forms. By modeling students expand the scope of algebraic operations to solve a wide variety of polynomial equations and real world problems. The role of factoring, as both an aid to the algebra and to the graphing of polynomials, is explored. Students identify zeros of polynomials, including complex zeros of quadratic polynomials, and make connections between zeros of polynomials and solutions of polynomial equations.

CLUSTERS	COMMON CORE STATE STANDARDS
(m) Create equations that describe numbers or relationships	**A-CED: Creating Equations★** **A-CED.1.** Create equations and inequalities in one variable including ones with absolute value and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. **CA ★** **A-CED.2.** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. ★ **A-CED.3.** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. **A-CED.4.** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. ★
(m) Represent and solve equations and inequalities graphically	**A-REI: Reasoning with Equations and Inequalities** **A-REI 1.11.** Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases

	where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. ★
(m) Perform arithmetic operations on polynomials	**A-A-APR: Arithmetic with Polynomials and Rational Expressions** **A-APR 1.** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
Understand the relationship between zeros and factors of polynomials	**A-APR 2.** Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x). **A-APR 3.** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
Use polynomial identities to solve problems	**A-APR 4.** Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.* **A-APR.5** Know and apply the Binomial Theorem for the expansion of (x + y)nin powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle. **(+)**
Perform arithmetic operations with complex numbers **Use complex numbers in polynomial identities and equations** *(Polynomials with real coefficients)*	**N-CN: The Complex Number System** **N-CN.1.** Know there is a complex number *i* such that $i^2 = -1$, and every complex number has the form $a + bi$ with *a* and *b* real. **N-CN.2.** Use the relation $i^2 = –1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. **N-CN.3.** Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. **(+)** **N-CN 7.** Solve quadratic equations with real coefficients that have complex solutions. **N-CN.8.** Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x – 2i)$. **(+)** **N-CN.9.** Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. **(+)**
MATHEMATICAL PRACTICES	
1. **Make sense of problems and persevere in solving them.** 2. **Reason abstractly and quantitatively.**	

3. Construct viable arguments and critique the reasoning of others. **4. Model with mathematics.** **5. Use appropriate tools strategically.** **6. Attend to precision.** **7. Look for and make use of structure.** **8. Look for and express regularity in repeated reasoning.**	Emphasize MP 1, 2, 3, 4, 5, 6, and 7 in this unit.

(m) Major Clusters – area of intensive focus where students need fluent understanding and application of the core concepts.

(S) Supporting/Additional Clusters – designed to support and strengthen areas of major emphasis/expose students to other subjects.

★Indicates a modeling standard linking mathematics to everyday life, work, and decision-making.

(+) Indicates additional mathematics to prepare students for advanced courses.

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS	KEY VOCABULARY
• Different types of relationships between quantities can be modeled with different types of functions. • Graphs are visual representations of solution sets of equations and inequalities. • The arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers. • Expressions that represent a quantity in terms of its context can be interpreted and its structure identified and rewritten. • The formula for the sum of a finite geometric series (when the common ratio is not 1) is derived and used to solve problems. • Polynomials form a system analogous to the integers which are closed under the operations of addition, subtraction, and multiplication and polynomial identities are proven to describe numerical relationships. • The Remainder Theorem can be applied for a polynomial p(x).	1. What relationships between quantities can be modeled by functions? 2. What does it mean to solve equations graphically? 3. What are the similarities and differences between linear, quadratic, and exponential functions? 4. What do extraneous solutions represent? 5. How does the arithmetic of rational numbers relate to simplifying rational expressions? 6. What does the graph of a function represent? 7. How can you represent the zeroes of a function? 8. How can you describe and show the ways you can find the zeroes (roots) of a function? 9. How can the formula for the sum of a finite geometric series be derived and used to solve problems? 10. How can you use the Binomial Theorem to expand powers of expressions? 11. What are the differences and similarities between real and complex solutions of polynomial equations? Explain graphically or algebraically.	• absolute value • binomial theorem • coefficient • complex numbers • complex roots • constraints • equations • equivalent • exponential • exponential • expressions • extraneous • factorization • factors • finite • function • functions • geometric series • inequalities

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS	KEY VOCABULARY
• Zeros of polynomials are identified when suitable factorizations are available and used to construct a rough graph of the function defined by the polynomial. • Binomial Theorem is for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers and known and applied. • Real and complex numbers are important in solving and understanding polynomial equations.		• infinite • interpret • linear • modeling • quadratic • quantities • radical equations • rational equations

RESOURCES	INSTRUCTIONAL STRATEGIES	ASSESSMENT
Materials: **California Revised Mathematics Framework:** http://www.cde.ca.gov/ci/ma/cf/documents/aug2013algebra2.pdf **Illustrative Mathematics:** Buying a Car: A-CED.1 http://www.illustrativemathematics.org/illustrations/582 Basketball: A-CED.1 & A-REI.2 http://www.illustrativemathematics.org/illustrations/702 How Much Folate: A-CED.2 http://www.illustrativemathematics.org/illustrations/1351 Dimes and Quarters: A-CED.2 & A-CED.3 http://www.illustrativemathematics.org/illustrations/220 Growing Coffee: A-CED.3 http://www.illustrativemathematics.org/illustrations/	Most standards in the Creating Equations domain carry a modeling star, denoting their connection with the Modeling category in high school. Therefore mathematical Modeling needs to be at the forefront of conversation with students. For example, equations in high school are also more likely to contain parameters that equations in earlier grades, and so interpreting a solution to an equation might involve more than consideration of a numerical value, but consideration of how the solution behaves as the parameters are varied. Provide examples of real-world problems that can be modeled by writing an equation or inequality. Begin with simple equations and inequalities and build up to more complex equations in two or more variables that may involve quadratic, exponential or rational functions. Give students examples of real-world problems that can be solved by writing an equation, and have	**Smarter Balanced Assessment Consortium (SBAC)** http://www.smarterbalanced.org/ **Partnership for Assessment of Readiness for Colleges and Careers (PARCC)** Seeing Structure in an Equation http://www.parcconline.org/samples/mathematics/high-school-seeing-structure-equation Seeing Structure in a Quadratic Equation http://www.parcconline.org/samples/mathematics/high-school-seeing-structure-quadratic-equation Graph of Functions http://www.parcconline.org/sites/parcc/files/HighSchoolAlg2Math3-GraphsofFunctions.pdf Brett's Race http://www.parcconline.org/sites/parcc/files/BRHSSampleItem.pdf

611 Bernardo and Sylvia Play a Game: A-CED.3 http://www.illustrativemathematics.org/illustrations/1010 Clea on an Escalator: A-CED.2 http://www.illustrativemathematics.org/illustrations/1003 Equations and Formulas: A-CED.4 http://www.illustrativemathematics.org/illustrations/393 Radical Equations: A-REI.2 http://www.illustrativemathematics.org/illustrations/391 Introduction to Polynomials - College Fund: A-REI.11 http://www.illustrativemathematics.org/illustrations/155	students explore the graphs of the equations using technology application to determine which parts of the graph are relevant to the problem context. Provide visual examples of radical and rational equations with technology so that students can see the solution as the intersection of two functions and further understand how extraneous solutions do not fit the model. Have students use technology to graph and explore functions. Discuss the meaning of parameters in the graph including the table, the curves, and the solution to the equation. Have students investigate real-world examples of two-dimensional inequalities. An instructional conversation with all students, in particular English learners will benefit from scaffolds that promote use of academic language. Mathematically Speaking is a scaffold that may be used. http://camsp.net/documents/NCTM-SpeakingArticle.pdf	

LANGUAGE GOALS

Writing:

1) Students will explain and justify the process of solving equations and inequalities by using key vocabulary terms.
Example: I solved for the variable in the inequality by _____. This means that _____.

2) Compare and contrast the differences and similarities between linear, quadratic, and exponential functions.
Example: The intercepts for linear graphs can be found by _____. The intercepts for quadratic functions can be found by _____. The intercepts for exponential functions can be found by _____.The intercept for the three types of graphs, are different because_____.

3) Students will explain how they developed their mathematical models.
Example: The variables in the equation represent _____. The terms and coefficients in the equations are added/ subtracted/multiplied/ divided because _____.

Listening and Speaking:

1) Students will generate class discussions using key vocabulary terms related to solving linear, quadratic, and exponential functions.
2) Students work in pairs to explain and justify how to solve an equation and summarize their partner's explanation using various tools, such as: media, poster, graphic organizer, etc.

Reading:

1) Students will identify mathematically relevant information from real-world scenarios and model equations with them.

PERFORMANCE TASKS

Mathematics Assessment Project

- Solving Linear Equations in Two Variables: A-CED.2
  http://map.mathshell.org/materials/lessons.php?taskid=209#task209
- Optimization Problems: Boomerangs: A-CED 2
  http://map.mathshell.org/materials/download.php?fileid=1241

Illustrative Mathematics

- Combined Fuel Efficiency: A-APR.6
  https://www.illustrativemathematics.org/illustrations/825
- Population and Food Supply: A-REI 11
  https://www.illustrativemathematics.org/illustrations/645

NCTM Illuminations

- Trout Pond Population: A-CED.2
  http://illuminations.nctm.org/Lesson.aspx?id=1549
- Exploring Linear Data: A-CED.2
  http://illuminations.nctm.org/Lesson.aspx?id=1189

DIFFERENTIATION

FRONT LOADING	ACCELERATION	INTERVENTION

• Involve students to have a discussion that center around extending their knowledge of creating and analyzing linear equations and inequalities. Have them use their prior knowledge of graphing linear equations and inequalities to solve real world scenarios. • Engage students in an activity that would involve comparing linear equations with quadratic equations, and then quadratic equations with exponential equations. • Have students match linear, quadratic, exponential functions with their graphs, tables, and equations.	• Provide examples of real-world problems that can be modeled using linear, polynomial, rational, absolute value, exponential, and logarithmic functions. Have students use technology to graph the functions and make tables of values. • Ask students to discover, model, and explain real-world scenarios in their everyday life that can be modeled using linear, polynomial, rational, absolute value, exponential, and logarithmic functions.	• Show students how to create numerical equations and then introduce linear equations in one variable. Students can make comparisons using numerical and linear equations. • Have students use technology to graph and generate tables of values for different types of equations. Lead student discussions about the graphs and tables of values to teach and reinforce key vocabulary terms such as intercepts, slopes, intersection, linear, roots, parabolas, etc…

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